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SYMMETRIES FOR VECTOR AND AXIAL-VECTOR MESONS*

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ABSTRACT

We discuss the possible symmetries of the effective lagrangian describing interacting pseudoscalar-, vector-, and axial-vector mesons. Specific choices of the parameters give rise to an $[SU(2) \otimes SU(2)]^3$ symmetry. This symmetry can be obtained either *à la* Georgi, with all states in the spectrum remaining massless, or in a new realization, implying decoupling of the pseudoscalar bosons from degenerate vector and axial-vector mesons. This second possibility, when minimally coupled with $SU(2) \otimes U(1)$ electroweak gauge vector bosons, seems particularly appealing. All deviations in the low-energy electroweak parameters are strongly suppressed, and the vector and axial-vector states turn out to be quite narrow. A nearby strong electroweak sector would then be conceivable and in principle it could be explored with relatively low energy accelerators.

1 Introduction

In this note we will discuss some properties of a low-energy effective theory of light pseudoscalar mesons and vector and axial-vector mesons. The obvious example of such a light bosonic spectrum is low-energy QCD. The other situation we have in mind, and to which the majority of our considerations will refer to, is dynamical breaking of the electroweak symmetry, with Goldstone bosons absorbed by the massive gauge vector bosons W and Z and a set of vector and axial-vector resonances. The effective theory can be built on the basis of the low-energy symmetry properties, using the well known techniques of CCWZ [1], considering the pseudoscalar mesons as Goldstone bosons of a spontaneously broken G group and the vector and axial-vector mesons as matter fields transforming under the unbroken subgroup H of G .

It has also appeared convenient at least formally, to describe the vector fields as gauge bosons of a spontaneously broken local symmetry H' , the so-called hidden symmetry [2][3]. In this description, the vector mesons acquire masses via a Higgs mechanism, by eating the would-be Goldstone bosons related to the breaking of H' . The explicit presence of these modes, which are absent in the traditional CCWZ construction, is the peculiar feature of the hidden gauge symmetry description. Within this approach, the symmetry group of the theory is enlarged to $G' = G \otimes H'$, with G and H' acting globally and locally, respectively. The invariance group of the vacuum is H_D , the diagonal subgroup of $H \otimes H'$, ($H' \supseteq H$), formally isomorphic to H .

Here we will focus our attention to the specific case: $G = SU(2)_L \otimes SU(2)_R$, $H' = SU(2)_L \otimes SU(2)_R$ and $H_D = SU(2)_V$, the latter being the diagonal $SU(2)$ subgroup of G' . The spontaneous breaking of G' down to H_D gives rise to nine Goldstone bosons. Six of them are eaten up by the vector and axial-vector mesons, triplets of $SU(2)_V$. The other three remain in the spectrum as massless physical particles (unless a part of the group G is promoted to a local symmetry). This particular situation has been discussed in ref. [2] for the QCD case and in ref. [4] in the context of dynamical electroweak symmetry breaking.

In the present note we would like to study in more detail the symmetry properties of the effective theory, pointing out that, in particular cases, the overall symmetry can be larger than the one requested by construction, namely that associated to the symmetry group G' . We shall find that a maximal symmetry $[SU(2) \otimes SU(2)]^3$ can be realized in the low energy effective lagrangian for pseudoscalar-, vector-, and axial-vector mesons for particular choices of the parameters. One choice naturally generalizes the vector-symmetry of Georgi, when axial-vector mesons are also included. This choice might offer some starting point for QCD calculations, but its eventual usefulness remains to be seen. A second choice will be the main object of our investigation. It may be useful in relation to strong electroweak breaking schemes and it has the peculiarity of allowing for a low energy strong electroweak resonant sector, at the same time satisfying the severe present experimental constraints, particularly from LEP data. In this sense such a choice appears of interest in view of its possible testing within existing or future machines of relatively low energy. These questions will be discussed below.

2 Vector-Axial Symmetries

We recall that the nine Goldstone bosons can be described by three independent $SU(2)$ elements: L , R and M , whose transformation properties with respect to G and H' are the following

$$L' = g_L L h_L, \quad R' = g_R R h_R, \quad M' = h_R^\dagger M h_L \quad (2.1)$$

where $g_{L,R} \in G$ and $h_{L,R} \in H'$. Beyond the invariance under G' , we will require also an invariance under the following discrete left-right transformation, denoted by P

$$P : \quad L \leftrightarrow R, \quad M \leftrightarrow M^\dagger \quad (2.2)$$

which also ensures that the low-energy theory is parity conserving.

Ignoring for a moment the transformation laws of eq. (2.1), the largest possible global symmetry of the low-energy theory is determined by the request of maintaining for the transformed variables L' , R' and M' , the character of $SU(2)$ elements. This selects as maximal symmetry the group $G_{max} = [SU(2) \otimes SU(2)]^3$, consisting of three independent $SU(2) \otimes SU(2)$ factors, acting separately on each of the three variables. Thus, it may happen that, for specific choices of the parameters characterizing the theory, the symmetry G' gets enlarged to G_{max} . In this note we will discuss two special cases which give rise to such a symmetry enhancement.

The most general $G' \otimes P$ invariant lagrangian is given by [4]

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{kin} \quad (2.3)$$

where

$$\mathcal{L}_G = -\frac{v^2}{4} [a_1 I_1 + a_2 I_2 + a_3 I_3 + a_4 I_4] \quad (2.4)$$

$$\begin{aligned} I_1 &= \text{tr}[(V_0 - V_1 - V_2)^2] \\ I_2 &= \text{tr}[(V_0 + V_2)^2] \\ I_3 &= \text{tr}[(V_0 - V_2)^2] \\ I_4 &= \text{tr}[V_1^2] \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} V_0^\mu &= L^\dagger D^\mu L \\ V_1^\mu &= M^\dagger D^\mu M \\ V_2^\mu &= M^\dagger (R^\dagger D^\mu R) M \end{aligned} \quad (2.6)$$

The covariant derivatives are defined by

$$\begin{aligned} D_\mu L &= \partial_\mu L - L \mathbf{L}_\mu \\ D_\mu R &= \partial_\mu R - R \mathbf{R}_\mu \\ D_\mu M &= \partial_\mu M - M \mathbf{L}_\mu + \mathbf{R}_\mu M \end{aligned} \quad (2.7)$$

and

$$\mathcal{L}_{kin} = \frac{1}{g''^2} tr[F_{\mu\nu}(\mathbf{L})]^2 + \frac{1}{g''^2} tr[F_{\mu\nu}(\mathbf{R})]^2 \quad (2.8)$$

where g'' is the gauge coupling constant for the gauge fields \mathbf{L}_μ and \mathbf{R}_μ , and

$$F_{\mu\nu}(\mathbf{L}) = \partial_\mu \mathbf{L}_\nu - \partial_\nu \mathbf{L}_\mu + [\mathbf{L}_\mu, \mathbf{L}_\nu] \quad (2.9)$$

In eq. (2.4) v represents a physical scale which depends on the particular context (QCD, electroweak symmetry breaking...) under investigation.

The quantities V_i^μ ($i = 0, 1, 2$) are, by construction, invariant under the global symmetry G and covariant under the gauge group H'

$$(V_i^\mu)' = h_L^\dagger V_i^\mu h_L \quad (2.10)$$

Out of the V_i^μ one can build six independent quadratic invariants, which reduce to the four I_i listed above, when parity is enforced.

For generic values of the parameters a_1, a_2, a_3, a_4 , the lagrangian \mathcal{L} is invariant under $G' \otimes P = G \otimes H' \otimes P$. There are however special choices which enhance the symmetry group. One of these is a special limit of the case $a_1 = 0, a_2 = a_3$. One has

$$\mathcal{L}_G = -\frac{v^2}{4} [2 a_2 tr(V_0)^2 + a_4 tr(V_1)^2 + 2 a_2 tr(V_2)^2] \quad (2.11)$$

Although the above choice of parameters has diagonalized the lagrangian in the variables L, R, M (see eq. (2.6)), this is not yet sufficient to modify the overall symmetry, due to the local character of the group H' which leads to a non-trivial dependence on the gauge fields \mathbf{L}_μ and \mathbf{R}_μ . However, if we consider the limit when these gauge interactions are turned off, then the lagrangian becomes

$$\mathcal{L}_G = \frac{v^2}{4} \{2 a_2 [tr(\partial_\mu L^\dagger \partial^\mu L) + tr(\partial_\mu R^\dagger \partial^\mu R)] + a_4 tr(\partial_\mu M^\dagger \partial^\mu M)\} \quad (2.12)$$

which is manifestly invariant under $G_{max} = [SU(2) \otimes SU(2)]^3$. This limit describes a set of nine massless, degenerate Goldstone bosons. It could be meaningful for vector and axial-vector mesons much lighter than the chiral breaking scale $4\pi v$ and almost degenerate with the pseudoscalar particles. This case can be considered as a generalization of the vector symmetry described by Georgi [5].

Another case of interest is provided by the choice: $a_4 = 0, a_2 = a_3$. To discuss the symmetry properties it is useful to observe that the invariant I_1 could be re-written as follows

$$I_1 = -tr(\partial_\mu U^\dagger \partial^\mu U) \quad (2.13)$$

with

$$U = LM^\dagger R^\dagger \quad (2.14)$$

and the lagrangian becomes

$$\mathcal{L}_G = \frac{v^2}{4} \{a_1 tr(\partial_\mu U^\dagger \partial^\mu U) + 2 a_2 [tr(D_\mu L^\dagger D^\mu L) + tr(D_\mu R^\dagger D^\mu R)]\} \quad (2.15)$$

Each of the three terms in the above expressions is invariant under an independent $SU(2) \otimes SU(2)$ group

$$U' = \omega_L U \omega_R^\dagger, \quad L' = g_L L h_L, \quad R' = g_R R h_R \quad (2.16)$$

Moreover, whereas these transformations act globally on the U fields, for the variables L and R , an $SU(2)$ subgroup is local. Different from the previous case, there is no need of turning the gauge interactions off. The overall symmetry is still $G_{max} = [SU(2) \otimes SU(2)]^3$, with a part H' realized as a gauge symmetry.

The field redefinition from the variables L , R and M to L , R and U has no effect on the physical content of the theory. It is just a reparametrization of the scalar manifold.

The extra symmetry related to the independent transformation over the U field, could be also expressed in terms of the original variable M . Indeed the lagrangian of eq. (2.4), for $a_4 = 0$, $a_2 = a_3$, possesses the additional invariance

$$L' = L, \quad R' = R, \quad M' = \Omega_R M \Omega_L^\dagger \quad (2.17)$$

with

$$\Omega_L = L^\dagger \omega_L L, \quad \Omega_R = R^\dagger \omega_R R \quad (2.18)$$

By expanding the lagrangian in eq. (2.4) in powers of the Goldstone bosons one finds, as the lowest order contribution, the mass terms for the vector mesons:

$$\mathcal{L}_G = -\frac{v^2}{4} [a_2 \text{tr}(\mathbf{L}_\mu + \mathbf{R}_\mu)^2 + (a_3 + a_4) \text{tr}(\mathbf{L}_\mu - \mathbf{R}_\mu)^2] + \dots \quad (2.19)$$

where the dots stand for terms at least linear in the Goldstone modes. By introducing the linear combinations

$$\mathbf{V}_\mu = \frac{1}{2}(\mathbf{R}_\mu + \mathbf{L}_\mu), \quad \mathbf{A}_\mu = \frac{1}{2}(\mathbf{R}_\mu - \mathbf{L}_\mu) \quad (2.20)$$

we can rewrite

$$\mathcal{L}_G = -v^2 [a_2 \text{tr}(\mathbf{V}_\mu)^2 + (a_3 + a_4) \text{tr}(\mathbf{A}_\mu)^2] + \dots \quad (2.21)$$

In the following we will focus on the case $a_4 = 0$, $a_2 = a_3$. Then, as we see from eq. (2.19), the mixing between \mathbf{L}_μ and \mathbf{R}_μ is vanishing, and the two states are degenerate in mass. Moreover, as it follows from eq. (2.15), the longitudinal modes of the \mathbf{L}_μ and \mathbf{R}_μ (or \mathbf{V}_μ , \mathbf{A}_μ) fields are entirely provided by the would-be Goldstone bosons in L and R . This means that the pseudoscalar particles remaining as physical states in the low-energy spectrum are those associated to U . They may in turn provide the longitudinal components to the W and Z particles, in an effective description of the electroweak breaking sector, or simply remain true Goldstone bosons if QCD is considered.

The peculiar feature of the limit under consideration is that these modes are decoupled from the vector and axial-vector mesons, as one can immediately deduce by inspecting eq. (2.15). In QCD this would mean that the coupling $g_{\rho\pi\pi}$ vanishes in the limit considered. The same remark also applies to the $g_{\rho a_1 \pi}$ coupling, which is absent. All this appears to be rather far from the real QCD picture. Nonetheless, it may not be unconceivable to model the low-energy strong interactions as a perturbation around the very crude approximation provided by the above scenario. The starting point would correspond to a symmetry, whose breaking could be systematically investigated by adding symmetry violating terms of increasing importance, in a chiral expansion. It would be of course of great interest to relate this symmetry to some particular limit of QCD.

3 Consequences for electroweak breaking

Let us now consider the coupling of the model to the electroweak $SU(2)_L \otimes U(1)_Y$ gauge fields as obtained via the minimal substitution

$$\begin{aligned} D_\mu L &\rightarrow D_\mu L + \mathbf{W}_\mu L \\ D_\mu R &\rightarrow D_\mu R + \mathbf{Y}_\mu R \\ D_\mu M &\rightarrow D_\mu M \end{aligned} \quad (3.1)$$

where

$$\begin{aligned} \mathbf{W}_\mu &= \frac{i}{2} g \tau_a W_\mu^a \\ \mathbf{Y}_\mu &= \frac{i}{2} g' \tau_3 Y_\mu \\ \mathbf{L}_\mu &= \frac{i}{2} \frac{g''}{\sqrt{2}} \tau_a L_\mu^a \\ \mathbf{R}_\mu &= \frac{i}{2} \frac{g''}{\sqrt{2}} \tau_a R_\mu^a \end{aligned} \quad (3.2)$$

with $a = 1, 2, 3$, and by introducing the canonical kinetic terms for W_μ^a and Y_μ . The factor $g''/\sqrt{2}$ in the definition of \mathbf{L}_μ and \mathbf{R}_μ comes from the re-scaling of the gauge fields to get canonical kinetic terms (see eq. (2.8)). The mass term for the vector mesons reads:

$$\mathcal{L}_G^{(2)} = -\frac{v^2}{4} [a_1 \text{tr}(\mathbf{W}_\mu - \mathbf{Y}_\mu)^2 + 2a_2 (\text{tr}(\mathbf{L}_\mu - \mathbf{W}_\mu)^2 + \text{tr}(\mathbf{R}_\mu - \mathbf{Y}_\mu)^2)] \quad (3.3)$$

The first term in the previous equation reproduces precisely the mass term for the ordinary gauge vector bosons in the SM, provided we identify the combination $v^2 a_1$ with $1/(\sqrt{2} G_F)$, G_F being the Fermi constant. Indeed, it is natural to think about the model we are considering as a perturbation around the SM picture. The SM relations are obtained in the limit $g'' \gg g, g'$. Actually, for a very large g'' , the kinetic terms for the fields \mathbf{L}_μ and \mathbf{R}_μ drop out, and $\mathcal{L}_G^{(2)}$ reduces to the first term in eq. (3.3). In the following we will assume:

$$a_1 = 1 \quad , \quad v^2 = 1/(\sqrt{2} G_F) \quad (3.4)$$

By writing $\mathcal{L}_G^{(2)}$ in terms of the charged and the neutral fields one finds:

$$\begin{aligned} \mathcal{L}_G^{(2)} &= \frac{v^2}{4} [(1 + 2a_2) g^2 W_\mu^+ W^{\mu-} + a_2 g''^2 (L_\mu^+ L^{\mu-} + R_\mu^+ R^{\mu-}) \\ &\quad - \sqrt{2} a_2 g g'' (W_\mu^+ L^{\mu-} + W_\mu^- L^{\mu+})] \\ &+ \frac{v^2}{8} [(1 + 2a_2) (g^2 W_3^2 + g'^2 Y^2) + a_2 g''^2 (L_3^2 + R_3^2) \\ &\quad - 2g g' W_{3\mu} Y^\mu - 2\sqrt{2} a_2 g'' (g W_3 L_3^\mu + g' Y_\mu R_3^\mu)] \end{aligned} \quad (3.5)$$

In the charged sector the fields R^\pm are completely decoupled from the remaining states, for any value of g'' . Their mass is given by:

$$M_{R^\pm}^2 = \frac{v^2}{4} a_2 g''^2 \quad (3.6)$$

On the contrary the modes L^\pm and W^\pm are mixed. By denoting with \hat{L}^\pm and \hat{W}^\pm the mass eigenstates, one obtains the following masses:

$$M_{\hat{W}^\pm}^2 = \frac{v^2}{4}g^2[1 - 2(\frac{g}{g''})^2 + \dots] \quad , \quad M_{\hat{L}^\pm}^2 = \frac{v^2}{4}a_2g''^2[1 + 2(\frac{g}{g''})^2 + \dots] \quad (3.7)$$

where the dots stand for higher order terms in g/g'' . The physical particle \hat{L}^\pm is a combination of L^\pm and W^\pm , which, for small values of g/g'' , is mainly oriented along the L^\pm direction.

In the neutral sector there is, as expected, a strictly massless combination which corresponds to the physical photon associated to the unbroken $U(1)$ gauge group. The remaining states, here denoted by Z , L_0 and R_0 , are massive and essentially aligned along the combinations $(c_\theta W_3 - s_\theta Y)$, L_3 and R_3 respectively (s_θ and c_θ denote the sine and the cosine of the Weinberg angle θ , with $\tan \theta = g'/g$). Unlike the charged case, however, the physical state R_0 is not completely decoupled, in fact at the leading order in g/g'' , it possesses a tiny component along the Y direction. The L_0 state has in turn a small contribution from the W_3 field. The non-vanishing masses are given by:

$$\begin{aligned} M_Z^2 &= \frac{v^2}{4} \frac{g^2}{c_\theta^2} [1 - \frac{1 + c_{2\theta}^2}{c_\theta^2} (\frac{g}{g''})^2 + \dots] \\ M_{L_0}^2 &= \frac{v^2}{4} a_2 g''^2 [1 + 2(\frac{g}{g''})^2 + \dots] \\ M_{R_0}^2 &= \frac{v^2}{4} a_2 g''^2 [1 + 2 \frac{s_\theta^2}{c_\theta^2} (\frac{g}{g''})^2 + \dots] \end{aligned} \quad (3.8)$$

where the dots represent higher order terms in the g/g'' expansion.

A non-trivial consequence of the mass spectrum of the model and of the structure of the bilinear terms given in eq. (3.5) concerns the low-energy effects which can be tested using the results of the available high-precision measurements. It has become customary to cast such analysis in the framework of the so-called ϵ variables [6] [7]. Alternatively one could use the analysis of Peskin and Takeuchi [8]. We recall that the deviations from the SM expectations for ϵ parameters are given by [7]

$$\begin{aligned} \delta\epsilon_1 &= e_1 - e_5 \\ \delta\epsilon_2 &= e_2 - s_\theta^2 e_4 - c_\theta^2 e_5 \\ \delta\epsilon_3 &= e_3 + c_\theta^2 e_4 - c_\theta^2 e_5 \end{aligned} \quad (3.9)$$

where we have kept into account the fact that in our case there are no vertex or box corrections to four-fermion processes, with the definitions

$$\begin{aligned} e_1 &= \frac{A_{33} - A_{WW}}{M_W^2} \\ e_2 &= F_{WW}(M_W^2) - F_{33}(M_Z^2) \\ e_3 &= \frac{c_\theta}{s_\theta} F_{30}(M_Z^2) \\ e_4 &= F_{\gamma\gamma}(0) - F_{\gamma\gamma}(M_Z^2) \\ e_5 &= M_Z^2 F'_{ZZ}(M_Z^2) \end{aligned} \quad (3.10)$$

The quantities A_{ij} and F_{ij} are related to the two-point vector boson functions

$$\Pi_{ij}^{\mu\nu}(q) = \Pi_{ij}(q^2)g^{\mu\nu} + (q^\mu q^\nu \text{ terms}) \quad (i, j = 0, 1, 2, 3) \quad (3.11)$$

by the decomposition:

$$\Pi_{ij}(q^2) = A_{ij} + q^2 F_{ij}(q^2) \quad (3.12)$$

In our case we get [9]

$$\begin{aligned} e_1 &= 0 \\ e_2 &= -s_\theta^2 \left(1 - \frac{M_Z^2}{M^2}\right) \left(1 - \frac{M_Z^2 c_\theta^2}{M^2}\right)^{-1} X \\ e_3 &= 0 \\ e_4 &= -\frac{s_{2\theta}^2}{2c_\theta^2} \left(1 - \frac{M_Z^2}{M^2}\right) X \\ e_5 &= \frac{c_\theta^4 + s_\theta^4}{c_\theta^2} X \end{aligned} \quad (3.13)$$

where

$$X = 2 \frac{M_Z^2}{M^2} \left(\frac{g}{g''}\right)^2 \frac{1}{\left(1 - \frac{M_Z^2}{M^2}\right)^2} \quad (3.14)$$

In the previous expressions M^2 denotes $v^2 a_2 g''^2 / 4$, the common squared mass of the vector bosons \mathbf{L}_μ and \mathbf{R}_μ , at leading order in the g/g'' expansion. In an expansion in M_Z^2/M^2 , the leading terms for the corrections to the epsilon parameters are then given by

$$\begin{aligned} \delta\epsilon_1 &= -\frac{c_\theta^4 + s_\theta^4}{c_\theta^2} X \\ \delta\epsilon_2 &= -c_\theta^2 X \\ \delta\epsilon_3 &= -X \end{aligned} \quad (3.15)$$

with $X \approx 2(M_Z^2/M^2)(g/g'')^2$. Notice that the deviations $\delta\epsilon$'s are negative. They are all of order X , which contains a double suppression factor: M_Z^2/M^2 and $(g/g'')^2$. The sum of the SM contributions, functions of the top and Higgs masses, and the previous deviations has to be compared with the experimental values for the ϵ parameters, determined from all the available low-energy data [10]

$$\begin{aligned} \epsilon_1 &= (3.5 \pm 1.7) \cdot 10^{-3} \\ \epsilon_2 &= (-5.4 \pm 4.7) \cdot 10^{-3} \\ \epsilon_3 &= (3.9 \pm 1.7) \cdot 10^{-3} \end{aligned} \quad (3.16)$$

Notice the relatively large error in the determination of ϵ_2 , mainly dominated by the uncertainty on the W mass. Indeed ϵ_1 provides the most stringent bound on the parameter X . Taking into account the SM value $(\epsilon_1)_{SM} = 3.8 \cdot 10^{-3}$, for $m_{top} = 174 \text{ GeV}$ and $m_H = 1000 \text{ GeV}$, we find the 90% CL limit:

$$M(\text{GeV}) \geq 1877 \left(\frac{g}{g''}\right) \quad (3.17)$$

leaving room for relatively light resonances beyond the usual SM spectrum.

We recall that the very small experimental value of the ϵ_3 parameter, which measures the amount of isospin-conserving virtual contributions to the vector boson self-energies, strongly disadvantages the ordinary technicolor schemes, for which the contribution to ϵ_3 is large and positive. This problem could be attributed to the vector dominance in the dispersion relation satisfied by the e_3 parameter (see eq. (3.10)) [8]:

$$e_3 = -\frac{g^2}{4\pi} \int_0^\infty \frac{ds}{s^2} [Im\Pi_{VV}(s) - Im\Pi_{AA}(s)] \quad (3.18)$$

where $\Pi_{VV(AA)}$ is the correlator between two vector (axial-vector) currents. If the vector and axial-vector spectral functions are saturated by lowest lying vector and axial-vector resonances, one has

$$Im\Pi_{VV(AA)}(s) = -\pi g_{V(A)}^2 \delta(s - M_{V(A)}^2) \quad (3.19)$$

where $g_{V(A)}$ parametrizes the matrix element of the vector (axial-vector) current between the vacuum and the state V_μ (A_μ), and $M_{V(A)}$ is the vector (axial-vector) mass. From the previous equations, one obtains

$$e_3 = \frac{g^2}{4} \left[\frac{g_V^2}{M_V^4} - \frac{g_A^2}{M_A^4} \right] \quad (3.20)$$

If the underlying theory mimics the QCD behaviour, naively scaled from $f_\pi \simeq 93 \text{ MeV}$ to $v \simeq 246 \text{ GeV}$, it follows that the contribution to ϵ_3 is unacceptably large [10]. On the contrary, in the present model the approximate degeneracy among the masses of the vector and axial-vector states and their couplings makes e_3 vanish. The ϵ_3 parameter is saturated by the remaining, small, e_4 and e_5 contributions (see eq. (3.9)).

The important feature of the model under examination is that the previous results are protected by the symmetry discussed in section 2. On the other hand we have tacitly assumed that, in the context of the electroweak symmetry breaking, this symmetry is explicitly broken only by terms of electroweak strength, originating by the minimal coupling given in eq. (3.1). Whether such an assumption is correct or not is a dynamical issue which could be answered only by analyzing the underlying fundamental theory. It would be of great interest to build up explicitly a more fundamental theory displaying in some limit an $[SU(2) \otimes SU(2)]^3$ symmetry, broken at low-energy by the electroweak interactions. On a more phenomenological level we may ask whether the low-energy phenomenology outlined above is modified by allowing breaking terms of more general form.

Coming back to the lagrangian of eqs. (2.3-2.5), with unspecified values of the a_i parameters, one gets [11]:

$$g_V = -\frac{1}{2}a_2 g'' v^2, \quad g_A = \frac{1}{2}a_3 g'' v^2 \quad (3.21)$$

Therefore, at the leading order in the weak interaction ($M_V^2 = a_2 g''^2 v^2/4$, $M_A^2 = (a_3 + a_4) g''^2 v^2/4$)

$$e_3 = \left(\frac{g}{g''} \right)^2 \left[1 - \frac{a_3^2}{(a_3 + a_4)^2} \right] \quad (3.22)$$

We see that $e_3 = 0$ is certainly guaranteed by assuming $g_V^2 = g_A^2$ and $M_V^2 = M_A^2$, which is the case for $a_2 = a_3$, $a_4 = 0$. If we apply a perturbation to this case respecting the $[SU(2) \otimes SU(2)]^2$ symmetry of \mathcal{L}_G , we see from eq. (3.22) that it produces an effect on e_3 only if $a_4 \neq 0$. In terms of the ϵ_3 parameter the unsuppressed part of $\delta\epsilon_3$ is controlled by a_4 :

$$\delta\epsilon_3 \approx 2 \frac{a_4}{a_3} \left(\frac{g}{g''} \right)^2 . \quad (3.23)$$

If $a_4 > 0$, the deviation for the ϵ_3 parameter is positive and potentially large, since it is no more suppressed by the double factor contained in X (see eq. (3.13-3.14)), which still protects the ϵ_1 and ϵ_2 parameters. This is what happens in the QCD-scaled technicolor models [11].

We conclude this section with some remarks about the decay of the vector mesons \mathbf{L}_μ and \mathbf{R}_μ . In the present, effective, description of the electroweak symmetry-breaking, the Goldstone bosons in U become unphysical scalars eaten up by the ordinary gauge vector bosons W and Z . The absence of coupling among U and the states L and R results in a suppression of the decay rate of these states into W and Z . Consider, for instance, the decay of the new neutral gauge bosons into a W pair. In a minimal model, with only vector resonances, this decay channel is largely the dominant one. The corresponding width is indeed given by [12]

$$\Gamma(V_0 \rightarrow WW) = \frac{G_F^2 M^5}{24\pi g''^2} \quad (3.24)$$

and it is enhanced with respect to the partial width into a fermion pair, by a factor $(M/M_W)^4$ [12]

$$\Gamma(V_0 \rightarrow \bar{f}f) \approx G_F M_W^2 \left(\frac{g}{g''} \right)^2 M . \quad (3.25)$$

This fact is closely related to the existence of a coupling of order g'' among V_0 and the unphysical scalars absorbed by the W boson. Indeed the fictitious width of V_0 into these scalars provides, via the equivalence theorem [13], a good approximation to the width of V_0 into a pair of longitudinal W and it is precisely given by eq. (3.24).

On the contrary, if there is no direct coupling among the new gauge bosons and the would-be Goldstone bosons which provide the longitudinal degree of freedom to the W , then their partial width into longitudinal W 's will be suppressed compared to the leading behaviour in eq. (3.24), and the width into a W pair could be similar to the fermionic width. As one can explicitly check, this entails that the trilinear couplings between the new gauge bosons and a W pair is no longer of order $g(g/g'')$, but at most of order $g(g/g'')^3$. From the trilinear kinetic terms and the mixing among the gauge bosons, we find, at the first non trivial order in (g/g'') :

$$\begin{aligned} g_{L_0 WW} &= \sqrt{2} \frac{g}{a_2} \left(\frac{g}{g''} \right)^3 \\ g_{L^\pm W^\pm Z} &= \frac{\sqrt{2}}{c_\theta} \frac{g}{a_2} \left(\frac{g}{g''} \right)^3 \\ g_{R_0 WW} &= \sqrt{2} \frac{g}{a_2} \tan^2 \theta \left(\frac{g}{g''} \right)^3 \end{aligned} \quad (3.26)$$

The R^\pm have no mixing whatsoever and therefore they will be absolutely stable as ensured by the phase invariance $R^\pm \rightarrow R^\pm \exp(\pm i\alpha)$. This means that $g_{R^\pm W^\pm Z} = 0$ at all order in (g/g'') . It may be useful to compare the couplings of \mathbf{L}_μ and \mathbf{R}_μ into vector boson pairs with those into fermions. If we do not introduce any direct coupling between the new vector bosons and the ordinary fermions, the only available interaction is the one obtained through the mixing of W^\pm , W_3 and Y with the new states. These couplings are described by the interaction lagrangian:

$$\mathcal{L}_F = -\sqrt{2}g \frac{g}{g''} J_{3L}^\mu L_{0\mu} + \sqrt{2}g \frac{g}{g''} \frac{s_\theta^2}{c_\theta^2} (J_{3L}^\mu - J_{em}^\mu) R_{0\mu} + g \frac{g}{g''} [J_\mu^+ L^{-\mu} + h.c.] \quad (3.27)$$

where J_μ^+ is the usual charged current, J_{3L}^μ and J_{em}^μ are the neutral currents related to the third component of the weak isospin and to the electromagnetic charge, respectively. Notice that also in this case the charged state R^\pm remains decoupled.

By using eqs. (3.26-3.27) we get the following expressions for the widths of the \mathbf{L}_μ and \mathbf{R}_μ bosons:

$$\Gamma(L_0 \rightarrow WW) = \Gamma(L^\pm \rightarrow W^\pm Z^0) = \frac{\sqrt{2}G_F M_W^2}{24\pi} M \left(\frac{g}{g''} \right)^2 \quad (3.28)$$

$$\Gamma(R_0 \rightarrow WW) = \frac{\sqrt{2}G_F M_W^2}{24\pi} \tan^4 \theta M \left(\frac{g}{g''} \right)^2 \quad (3.29)$$

$$\Gamma(L_0 \rightarrow \nu\bar{\nu}) = \frac{1}{2}\Gamma(L^+ \rightarrow e^+\nu) = \frac{\sqrt{2}G_F M_W^2}{12\pi} M \left(\frac{g}{g''} \right)^2 \quad (3.30)$$

$$\Gamma(R_0 \rightarrow \nu\bar{\nu}) = \frac{\sqrt{2}G_F M_W^2}{12\pi} \tan^4 \theta M \left(\frac{g}{g''} \right)^2 \quad (3.31)$$

which confirm the expected behaviour, linear in the heavy mass M .

4 Conclusions

We have discussed the symmetry properties of an effective lagrangian describing light pseudoscalars interacting with vector and axial-vector mesons. The maximal symmetry of such system, $[SU(2) \otimes SU(2)]^3$, is exhibited by specific choices of the parameters characterizing the model.

One of these choices represents the natural generalization for including axial-vectors in the so-called vector symmetry, proposed by Georgi in the context of the strong interactions. In its realization the spectrum consists of a set of nine massless Goldstone bosons, six of which representing the longitudinal degrees of freedom of the vector and axial-vector mesons. This description could be meaningful for light vector and axial-vector states, almost degenerate with the pseudoscalar particles. Its eventual usefulness for QCD remains to be investigated.

A second possible realization of the maximal symmetry, which we have discussed in more detail, is instead characterized by a degeneracy among the vector and axial-vector

particles which, however, are no longer bound to have the same mass of the pseudoscalar modes. The latter, in the symmetry limit, are completely decoupled from the vector meson states. By minimally coupling this model to the electroweak vector bosons W_μ^a and Y_μ , we obtain an effective model for the electroweak symmetry breaking, in which the $[SU(2) \otimes SU(2)]^3$ symmetry is broken by the weak interaction terms. The physical features of such model are quite peculiar. They are tightly related to the special breaking of $[SU(2) \otimes SU(2)]^3$ considered, limited to terms originating from the minimal coupling to the electroweak vector bosons. Additional breaking terms, as allowed in general by the hidden symmetry construction, would lead to substantial modifications.

In our vector-axial degenerate model all the low-energy effects are strongly suppressed, unlike the usual technicolor-inspired effective models. In particular, the tiny deviations in the ϵ parameters are negative and of order M_Z^4/M^4 , M denoting the common mass of the vector mesons. This leaves room for relatively light vector resonances. Moreover, due to the vanishing of the coupling between the vector mesons and the would-be Goldstone modes absorbed by W and Z , the decay widths of the vector and axial-vector states are equally shared among the vector boson pair and fermion channels. Such widths, being of order M_Z/M times the typical Z fermionic width, are naturally quite small.

References

- [1] S.Coleman, J.Wess and B.Zumino, Phys. Rev. **177** (1969) 2239;
C.G.Callan, S.Coleman, J.Wess and B.Zumino, Phys. Rev. **177** (1969) 2247
- [2] M.Bando, T.Kugo and K.Yamawaki, Phys. Rep. 164 (1988) 217
- [3] A.P.Balachandran, A.Stern and G.Trahern, Phys. Rev. **D19** (1979) 2416
- [4] R.Casalbuoni, S.De Curtis, D.Dominici F.Feruglio and R.Gatto, Int. Jour. Mod. Phys. **A4** (1989) 1065
- [5] H.Georgi, Nucl. Phys. **B331** (1990) 311
- [6] G.Altarelli and R.Barbieri, Phys. Lett. 253B (1991) 161;
G.Altarelli, R.Barbieri and S.Jadach, Nucl. Phys. B369 (1992) 3;
G.Altarelli, R.Barbieri and F.Caravaglios, Nucl. Phys. B405 (1993) 3
- [7] R.Barbieri, F.Caravaglios and M.Frigeni, Phys. Lett. B279 (1992) 169
- [8] M.E.Peskin and T.Takeuchi, Phys. Rev. Lett. **65** (1990) 964 and Phys. Rev. **D46** (1991) 381
- [9] R.Casalbuoni, S.De Curtis, D.Dominici, F.Feruglio and R.Gatto, Phys. Lett. **B258** (1991) 161
- [10] G.Altarelli, CERN Preprint, CERN-TH 7464/94 (1994)
- [11] R.Casalbuoni, A.Deandrea, S.De Curtis, N.Di Bartolomeo, D.Dominici, F.Feruglio and R.Gatto, Nucl. Phys. **B409** (1993) 257

- [12] R.Casalbuoni, P.Chiappetta, A. Deandrea, S.De Curtis, D.Dominici, F.Feruglio and R.Gatto, Z. für Physik **C60** (1993) 315
- [13] J.M.Cornwall, D.N.Levine and G.Tiktopoulos, Phys. Rev. **D10** (1974) 1145;
C.G.Vayonakis, Lett. Nuovo Cimento **17** (1976) 17;
M.S.Chanowitz and M.K.Gaillard, Nucl. Phys. **B261** (1985) 379;
G.J.Gounaris, R.Kögerler and H.Neufeld, Phys. Rev. **D 34** (1986) 3257.